Numerical simulations of $N=(1,1)$ 1+1-dimensional super Yang-Mills theory with large supersymmetry breaking

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We consider the $N=(1,1)$ super Yang-Mills (SYM) theory that is obtained by dimensionally reducing SYM theory in 2+1 dimensions to 1+1 dimensions and discuss soft supersymmetry breaking. We discuss the numerical simulation of this theory using supersymmetric discrete light-cone quantization when either the boson or the fermion has a large mass. We compare our result to the pure adjoint fermion theory and pure adjoint boson discrete light-cone quantization calculations of Klebanov, Demeterfi, Bhanot and Kutasov. With a large boson mass we find that it is necessary to add additional operators to the theory to obtain sensible results. When a large fermion mass is added to the theory we find that it is not necessary to add operators to obtain a sensible theory. The theory of the adjoint boson is a theory that has stringy bound states similar to the full SYM theory. We also discuss another theory of adjoint bosons with a spectrum similar to that obtained by Klebanov, Demeterfi, and Bhanot.

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I. INTRODUCTION

In recent years we have worked extensively on a numerical method [1] for solving exactly supersymmetric field theories in $1+1$ and $2+1$ dimensions. We call this method supersymmetric discrete light-cone quantization (SDLCQ), and we have successfully applied it to many theories and addressed a number of interesting issues [2–9]. The world is, however, not exactly supersymmetric, and it is therefore important to learn how to generalize SDLCQ to solve nonsupersymmetric theories.

With this objective in mind we focus in this work on the interrelation of the numerical simulations of four separate theories. Two of these theories are supersymmetric, and two are not. The objective is to learn something about soft supersymmetry breaking within the context of these numerical simulations. We want to know if softly broken theories make sense in the context of SDLCQ and how these broken theories are related to nonsupersymmetric theories of adjoint fermions and adjoint bosons that have been discussed in the literature.

The two nonsupersymmetric simulations are those of Klebanov, Demeterfi and Bhanot [10,11] (KDB). The first is a calculation of a gauged adjoint fermion in 1+1 dimensions and the second is a calculation of a gauged adjoint boson in $1+1$ dimensions. The third theory we will consider is again a gauged adjoint fermion theory in 1+1 dimensions but now with a special mass value that makes the theory supersymmetric [12]. Finally, the fourth theory is the $N=(1,1)$ super Yang-Mills (SYM) theory that one obtains by dimensionally reducing SYM theory in 2+1 dimensions to 1+1 dimensions [1,2]. Henceforth we will refer to this theory as SYM theory. Starting with this fourth theory we consider the theories that are obtained by adding a large mass for one of the fields. The large mass freezes out that field, leaving the low-mass bound states to have primarily only constituents of the other field.

The numerical method that we use to simulate these theories is discrete light-cone quantization (DLCQ). When this method is applied to the continuum Hamiltonian of a theory, it is called just DLCQ, and it produces a finite dimensional Hamiltonian. When this method is applied to the continuum supercharge of a supersymmetric theory, it produces a finite dimensional supercharge which is then used to calculate a finite dimensional Hamiltonian, $P^-=(Q^-)^2/\sqrt{2}$. We refer to this approximation as supersymmetric DLCQ or SDLCQ.

To discretize the Hamiltonian or supercharge, we introduce discrete longitudinal momenta k^+ as fractions nP^+/K of the total longitudinal momentum P^+, where K is an integer that determines the resolution of the discretization and is known in DLCQ as the harmonic resolution [13]. We then convert the mass eigenvalue problem $2P^+P^-|M\rangle=M^2|M\rangle$ $(P^\pm=(P^z\pm P^0)/\sqrt{2})$ to a matrix eigenvalue problem by introducing a basis where P^+ is diagonal. Because light-cone longitudinal momenta are always positive, K and each n are positive integers; the number of constituents is then bounded by K. The continuum limit is recovered by taking the limit $K\to\infty$.

Of course, we can write the continuum Hamiltonian for a supersymmetric theory and apply DLCQ to it. This yields a different finite dimensional approximation to the Hamiltonian than does SDLCQ. Recently we have developed a technique for writing down directly finite-dimensional DLCQ Hamiltonians that are identical to the Hamiltonians obtained in SDLCQ [9].

In constructing the discrete approximation we drop the longitudinal zero-momentum mode. For some discussion of dynamical and constrained zero modes, see the review [13] and previous work [2]. Inclusion of these modes would be ideal, but the techniques required to include them in a numerical calculation have proved to be difficult to develop, particularly because of nonlinearities. For DLCQ calculations that can be compared with exact solutions, the exclusion of zero modes does not affect the massive spectrum [13]. In scalar theories it has been known for some time that constrained zero modes can give rise to dynamical symmetry breaking [13].

In Sec. II we discuss SYM theory in both the SDLCQ and DLCQ formulation. We consider in some detail the singular terms in the Hamiltonian formulation and their action on the Fock basis. In Sec. III we add a large boson mass to this theory and discuss the resulting pure fermion theory. We compare our results to the fermionic theories of KDB and Kutasov [10–12]. In Sec. IV we add a large fermionic mass to the full SYM theory and discuss the pure boson theory that is obtained. We find a new theory of adjoint bosons with properties similar to the full SYM theory. We consider the behavior of this theory with a small bare mass term for the boson and the relation of this theory to the KDB theory of adjoint bosons. We also present yet another theory similar to the KDB theory that arises from the discussion. In Sec. V we summarize the results of this investigation of supersymmetry breaking and the implication for higher dimensional theories.

II. THE SUPERSYMMETRIC THEORY

The SYM theory is given by the supercharge [1]

$$Q^-=\frac{i2^{-1/4}g}{\sqrt{\pi}}\int_0^\infty dk_1dk_2dk_3\delta(k_1+k_2-k_3)\Bigg\{\frac{1}{2\sqrt{k_1k_2}}\frac{k_2-k_1}{k_3}[a^\dagger_{ik}(k_1)a^\dagger_{kj}(k_2)b_{ij}(k_3)-b^\dagger_{ij}(k_3)a_{ik}(k_1)a_{kj}(k_2)]$$
$$\times\frac{1}{2\sqrt{k_1k_3}}\frac{k_1+k_3}{k_2}[a^\dagger_{ik}(k_3)a_{kj}(k_1)b_{ij}(k_2)-a^\dagger_{ik}(k_1)b^\dagger_{kj}(k_2)a_{ij}(k_3)]\frac{1}{2\sqrt{k_2k_3}}\frac{k_2+k_3}{k_1}$$
$$\times[b^\dagger_{ik}(k_1)a^\dagger_{kj}(k_2)a_{ij}(k_3)-a^\dagger_{ij}(k_3)b_{ik}(k_1)a_{kj}(k_2)]\left(\frac{1}{k_1}+\frac{1}{k_2}-\frac{1}{k_3}\right)[b^\dagger_{ik}(k_1)b^\dagger_{kj}(k_2)b_{ij}(k_3)+b^\dagger_{ij}(k_3)b_{ik}(k_1)b_{kj}(k_2)]\Bigg\}, \tag{1}$$

which in turn defines the Hamiltonian by virtue of the anticommutation relation $\{Q^-,Q^-\}=2\sqrt{2}P^-$. Throughout this paper we will write expressions in a continuum form for notational convenience; however, it is to be understood that all the calculations are discrete in momentum space. The numerical method SDLCQ simply means that we apply DLCQ to the supercharge and then square the finite dimensional representation of the supercharge to get the Hamiltonian. Recently we found the Hamiltonian that in the DLCQ approximation reproduces the SDLCQ Hamiltonian

$$P^-=\frac{g^2N_c}{4\pi}\int_0^\infty dk_1\frac{\mu^2(k_1)}{k_1}(a^\dagger a+b^\dagger b)+\frac{g^2}{4\pi}\int_0^\infty dk_1dk_2dk_3dk_4[A_1b^\dagger b^\dagger bb+A_2(b^\dagger bbb-b^\dagger b^\dagger b^\dagger b)+B_1a^\dagger a^\dagger aa$$
$$+B_2(a^\dagger aaa+a^\dagger a^\dagger a^\dagger a)+C_1b^\dagger b^\dagger aa+C_2a^\dagger a^\dagger bb+C_3b^\dagger a^\dagger ba+C_4a^\dagger b^\dagger ab+C_5b^\dagger a^\dagger ab+C_6a^\dagger b^\dagger ba$$
$$+D_1(a^\dagger abb-a^\dagger b^\dagger b^\dagger a)+D_2(a^\dagger bab-b^\dagger a^\dagger b^\dagger a)+D_3(a^\dagger bba-b^\dagger b^\dagger a^\dagger a)+D_4(b^\dagger baa+b^\dagger a^\dagger a^\dagger b)$$
$$+D_5(b^\dagger aba+a^\dagger b^\dagger a^\dagger b)+D_6(b^\dagger aab+a^\dagger a^\dagger b^\dagger b)], \tag{2}$$

where the coefficient in front of the dynamic mass term is given by [9]

$$\mu^2(k_1)=\int_0^{k_1}dk_2\frac{(k_1+k_2)^2}{k_2(k_1-k_2)^2}=\int_0^{k_1}dk_2\left(\frac{4k_1}{(k_2-k_1)^2}+\frac{1}{k_2}\right). \tag{3}$$

It is convenient to define the instantaneous mass contributions to the Hamiltonian, $P^-_{I\text{mass}}(\text{boson})$ and $P^-_{I\text{mass}}(\text{fermion})$, as

$$P^-_{I\text{mass}}(\text{boson})=\frac{g^2N_c}{\pi}\int_0^\infty dk_1a(k_1)^\dagger a(k_1)\int_0^{k_1}\frac{dk_2}{(k_2-k_1)^2},$$

$$P^-_{I\text{mass}}(\text{fermion})=\frac{g^2N_c}{\pi}\int_0^\infty dk_1b(k_1)^\dagger b(k_1)\int_0^{k_1}\frac{dk_2}{(k_2-k_1)^2}, \tag{4}$$

which are part of μ defined above. The coefficients of the pure fermion and the pure boson terms are

$$A_1=PV\frac{2}{(k_4-k_2)^2}-\frac{2}{(k_1+k_2)^2}-\delta_{1,3}\left(\frac{2}{k_1^2}+\frac{2}{k_2^2}\right),$$

$$A_2=\frac{2}{(k_2+k_3)^2}-\frac{2}{(k_1+k_2)^2},$$

$$B_1=\frac{1}{\sqrt{4k_1k_2k_3k_4}}\left(\frac{(k_1-k_2)(k_3-k_4)}{(k_1+k_2)^2}-PV\frac{(k_1+k_3)(k_2+k_4)}{(k_4-k_2)^2}\right),$$

$$B_2=\frac{1}{\sqrt{4k_1k_2k_3k_4}}\left(\frac{(k_3-k_2)(k_1+k_4)}{(k_3+k_2)^2}+\frac{(k_1-k_2)(k_3+k_4)}{(k_1+k_2)^2}\right). \tag{5}$$

The term $\delta_{1,3}$ only makes sense in a discrete formulation and vanishes in the continuum limit; however, it does not appear in a conventional DLCQ formulation. This term arises from normal ordering the square of the discrete supercharges rather than normal ordering the continuum formulation of the supercharge and then discretizing this result. These terms are part of what insures that this Hamiltonian is exactly supersymmetric. A detailed discussion of the origin of these terms can be found in [9]. Similarly, when obtaining the SDLCQ Hamiltonian version of the Kutasov model one finds for the same reason a different discrete formula for the mass term than in conventional DLCQ.

The singularities in $P_{I\text{mass}}$ cancel the singularities in A_1 and B_1. This cancellation is commonly seen in light-cone calculations and is not related to supersymmetry. We demonstrated that this occurs for both the pure fermionic and pure bosonic theories by considering the effect of these terms on Fock states. We will see that KDB in their theory of adjoint bosons treat these singularities differently than they are treated in SDLCQ and in the KDB and Kutasov treatment of adjoint fermions.

The $1/k_2$ term in μ is a real logarithmic mass divergence. In a nonsupersymmetric theory this requires a mass renormalization. Here in a supersymmetric theory the bound states are such that this term is finite.

It is interesting to compare this to the QCD_2 model described by Kutasov [12], where

$$Q^-=\frac{i2^{-1/4}g}{\sqrt{\pi}}\int_0^\infty dk_1dk_2dk_3\,\delta(k_1+k_2-k_3)\times\left(\frac{1}{k_1}+\frac{1}{k_2}-\frac{1}{k_3}\right)[b^\dagger_{ik}(k_1)b^\dagger_{kj}(k_2)b_{ij}(k_3)+b^\dagger_{ij}(k_3)b_{ik}(k_1)b_{kj}(k_2)]. \tag{6}$$

From normal ordering the discrete supercharge we find that the Hamiltonian for this theory contains the terms A_1, A_2, $P^-_{I\text{mass}}$(fermion) and the mass term with $m^2=g^2N_c/\pi$. We note that it does not contain the real logarithmic divergence seen in SYM theory.

III. PURE ADJOINT FERMION THEORIES

The pure adjoint fermion theories that we wish to consider are the DLCQ Hamiltonian theory studied by KDB [10], the supersymmetric model of Kutasov [12], and the pure fermion theory one obtains by including a large mass for the adjoint bosons in SYM theory. Let us start by considering the effect of A_1 and $P^-_{I\text{mass}}$ on a typical Fock state

$$|bbb\rangle=\int_0^\infty\frac{dsdndk}{N^{3/2}}f(s,n,k)\times\mathrm{Tr}[b^\dagger(s)b^\dagger(n)b^\dagger(k)]|0\rangle. \tag{7}$$

We find

$$P^-_{I\text{mass}}|bbb\rangle=\frac{3g^2N}{2\pi}\int_0^\infty\frac{dsdndk}{N^{3/2}}\times\delta(s+n+k-P^+)\times\mathrm{Tr}[b^\dagger(s)b^\dagger(n)b^\dagger(k)]|0\rangle\times\int_0^{s+n}\frac{dt}{(s-t)^2}f(s,n,k). \tag{8}$$

The factor of three is from using the cyclic symmetry to combine the action on all three permutations. The effect of the singular term in A_1 on this Fock state is

$$P^-_{\text{sing}}|bbb\rangle=\frac{-3g^2N}{2\pi}\int_0^\infty\frac{dsdndk}{N^{3/2}}\delta(s+n+k-P^+)\times\mathrm{Tr}[b^\dagger(s)b^\dagger(n)b^\dagger(k)]|0\rangle\times\int_0^{s+n}\frac{dt}{(s-t)^2}f(t,s+n-t,k). \tag{9}$$

Again the factor of three is from using the cyclic symmetry to combine the action on all three permutations. We see that the singularities in these terms at $s=t$ cancel, leaving an integral that is well defined and finite in the principal value prescription. In Ref. [10] the authors explicitly consider the integral equation for the Hamiltonian of adjoint fermions and find this cancellation. Numerically all the theories involving adjoint fermions handle this singularity the same way. They include both terms and omit the discrete point $s=t$.

It is already known that the KDB [10] and Kutasov [12] formulations produce the same numerical results for a bare mass squared of g^2N_c/π in the continuum limit. We want to consider the SYM theory and add a larger adjoint boson mass term ($x=m^2_{\text{boson}}\pi/g^2N$) to the square of the supercharge, which will freeze out the bosons. Of course, if we also subtract from the square of the supercharge the logarithmically divergent fermion mass term and add a bare mass term with $m^2=g^2N_c/\pi$, we reproduce the results of the Kutasov model. For completeness we have checked this, and it works exactly for bound states well below the mass of the heavy boson.

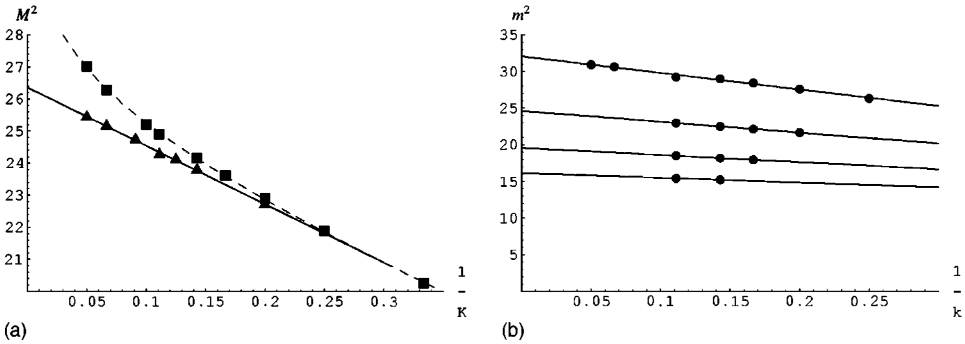

FIG. 1. $\pi M^2/g^2N_c$ vs $1/K$, where K is the resolution, for SYM theory with (a) a large boson mass and (b) a very large fermion mass. In (a), the triangles represent the Kutasov model, for comparison.

The main question that we consider in this section is whether the theory we obtain by simply adding a large mass for the boson is a sensible theory. We have looked at this numerically and the answer is no. It produces negative-mass bound states that do not disappear as we increase the resolution. We can repair this problem by adding a bare mass for the adjoint fermion, but this does not guarantee a sensible theory. For example, if we add a mass of $m^2=g^2N_c/\pi$, we effectively have the Kutasov model with a logarithmically divergent mass term included. We expect this model to produce bound-state masses that diverge logarithmically. In Fig. 1(a) we have plotted the lowest bound states of this theory and of the Kutasov model. The fit to the mass of this theory is

$$M^2=\frac{g^2N_c}{\pi}\left[23.22-13.66\frac{1}{K}-1.48\log(K)\right]. \qquad (10)$$

Therefore the pure fermionic theories that are obtained by adding a large boson mass require a mass renormalization exactly as we expected.

Within the context of SDLCQ we have also looked at the decoupling of the heavy bosonic states in more detail. At resolution $K=4$, for example, we can look at the evolution of the lowest-mass bound states as a function of the boson mass x in units of g^2N_c/π. At this resolution, the pure fermionic state $b^\dagger(3)b^\dagger(1)$ mixes with $a^\dagger(3)a^\dagger(1)$ and $a^\dagger(2)a^\dagger(2)$. The corresponding Hamiltonian matrix for the set of states $\{b^\dagger(3)b^\dagger(1),a^\dagger(3)a^\dagger(1),a^\dagger(2)a^\dagger(2)\}$ is

$$\begin{pmatrix} 21.88 & -0.96 & 1.41 \\ -0.96 & 8.33+5.33x & -12.24 \\ 1.41 & -12.24 & 18+4x \end{pmatrix}.$$

At large x the lowest mass eigenstate is $\{1,\theta_1,\theta_2\}$, where θ_1 and θ_2 are the bosonic components. We find that θ_1 and θ_2 fall off linearly with $1/x$.

IV. PURE ADJOINT BOSON THEORIES

The pure boson theories that we want to compare are the theory of KDB and SYM theory with a large mass for the adjoint fermion. Physically the bosons in these theories are the transverse gluons of the (2+1)-dimensional parent theory; therefore, these states are effectively (1+1)-dimensional glueballs.

First we consider the result of simply adding a large fermion mass to the SYM theory. This leaves us with the pure boson theory which contains the $P^-_{I\text{mass}}$ term and the logarithmically divergent mass term, just as in the fermion theory. Recall that the fermion theory with the logarithmically divergent mass had a divergent spectrum, but surprisingly the low-mass states of the boson theory, seen in Fig. 1(b), are linear as functions of $1/K$, and therefore the continuum spectrum obtained by extrapolating to $K=\infty$ appears to remain finite. We have calculated one state out to resolution $K=20$, to check for any logarithmic dependence, and found none. A partial explanation for this has to lie in the stringy nature of these bound states. In the spectrum of the pure SYM theory we found that, as we increased the resolution, new bound states appeared with more partons and with a lower mass. This abundance of low-mass states with many partons is what we refer to as the stringy nature of the theory. Interestingly we see this property for this pure glue theory here in Fig. 1(b). The fact that the logarithmically divergent mass term does not give rise to a divergent spectrum as a function of the resolution is apparently related to the stingy nature of the bound states.

We now want to contrast this stringy theory with the model considered by KDB [11]. There are two main differences between these two models. First, KDB renormalizes away the logarithmically divergent mass term. Second, they use a different approach to treat the singularity in the $P^-_{I\text{mass}}$ term and the singular term in B_1. This issue is a little complicated, so let us start our discussion by considering the effect of these two terms on a typical Fock state

$$|aaa\rangle=\int_0^\infty\frac{dsdndk}{N^{3/2}}f(s,n,k)\times\mathrm{Tr}[a^\dagger(s)a^\dagger(n)a^\dagger(k)]|0\rangle. \qquad (11)$$

We find

$$P^-_{I\text{mass}}(\text{boson})|aaa\rangle=\frac{3g^2N}{2\pi}\int_0^\infty\frac{dsdndk}{N^{3/2}}\delta(s+n+k-P^+)\times\mathrm{Tr}[a^\dagger(s)a^\dagger(n)a^\dagger(k)]|0\rangle\times\int_0^{s+n}\frac{dt}{(n-t)^2}f(s,n,k). \qquad (12)$$

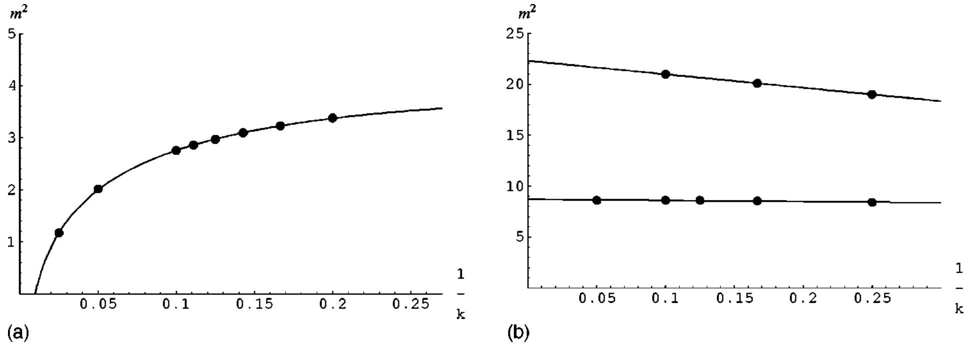

FIG. 2. $\pi M^2/g^2N_c$ vs $1/K$, where K is the resolution, for the SDLCQ solution of SYM theory with addition of a heavy fermion mass, subtraction of the logarithmically divergent boson mass, and addition of a bare mass of (a) $m^2=g^2N_c/\pi$ or (b) $m^2=2g^2N_c/\pi$.

This is, of course, the same as the result for adjoint fermions. The singular interaction term in B_1 gives

$$P^-_{\rm sing}|aaa\rangle = \frac{-3g^2N}{4\pi}\int_0^\infty \frac{ds\,ds\,dt\,dk}{N^{3/2}}\,\delta(s+n+k-P^+) \times {\rm Tr}[a^\dagger(s)a^\dagger(n)a^\dagger(k)]|0\rangle \times\int_0^{s+n} dt\,\frac{(t+n)(n+2s-t)f(t,s+n-t,k)}{2\sqrt{nst(n+s-t)}\,(n-t)^2}. \tag{13}$$

This expression has a complicated coefficient not present for fermions. We see that at the singular point $n=t$ the two terms cancel; therefore, the combination is nonsingular in the principal value prescription. Numerically, in the SDLCQ calculation, these two terms are simply included in their discrete form with the singular points removed. This exactly parallels the adjoint fermion calculations discussed above.

KDB treat the singularities differently, however. To regularize the singularity in B_1 they add and subtract a term

$$\frac{-3g^2N}{4\pi}\int_0^\infty \frac{ds\,ds\,dt\,dk}{N^{3/2}}\,\delta(s+n+k-P^+) \times {\rm Tr}[a^\dagger(s)a^\dagger(n)a^\dagger(k)]|0\rangle \times f(s,n,k)\int_0^{s+n} dt\,\frac{(t+n)(n+2s-t)}{2\sqrt{nst(n+s-t)}\,(n-t)^2}. \tag{14}$$

The part that is subtracted makes Eq. (13) finite in the principal value sense. For the part that is added they cut out the singular point and do the remaining principal value integral exactly and include it in the integral equation. The singular part cancels exactly the singularity in $P_{I\rm mass}$.

The remaining finite part of $P_{I\rm mass}$ has the form of a mass term with $m^2=-2g^2N/\pi$. They lump this term together with the logarithmically divergent mass term and a bare mass term, to form a renormalized mass term. In principle this appears to be simply a different way of renormalizing the mass and a different way of making the singularity finite in the principal value prescription.

Numerically KDB find a very different spectrum than we found above. Their spectrum is very QCD-like, with the lowest-mass bound states having primarily two gluons and masses of about $4g^2N_c/\pi$. The higher mass states have dominant components with a small number of particles. KDB did the calculation using antiperiodic boundary conditions, and we have repeated the calculation using both periodic and antiperiodic boundary conditions. We get the same results by both methods, and our antiperiodic calculation agrees exactly with KDB. The convergence is similar in both methods when one takes into account the fact the you have to go to twice the resolution with antiperiodic boundary conditions to get the same number of data points.

It is interesting to make a direct comparison of the KDB approach with SDLCQ. To make this comparison we will repeat the SDLCQ calculation, but now we will drop the divergent mass term and add a bare mass term as KDB do and then calculate the spectrum for various values of the bare mass. With the bare mass equal to zero the theory is unstable and generates a negative mass for the bound states. For $m^2=g^2N_c/\pi$ the mass of the lowest bound state as a function of the resolution is shown in Fig. 2(a). It clearly does not converge well as a function of $1/K$. It is possible that in the continuum limit the mass of this state goes to zero, but it also possible that it becomes negative. The fit shown is

$$M^2=\frac{g^2N_c}{\pi}\left[6.00-2.80\frac{1}{K}+1.29078\log(K)\right]. \tag{15}$$

The lowest-mass bound state for $m^2=2g^2N_c/\pi$ is shown in Fig. 2(b), and we see that the bound state is well behaved and is fit nicely by a linear plot in $1/K$. In fact the entire spectrum for this theory is well behaved. An inspection of the wave functions of this theory shows that it has a valence structure similar to KDB but not identical.

V. DISCUSSION

We have considered the effect of adding a large boson or fermion mass to SYM theory and compared the results with those known for pure fermionic and pure bosonic theories discussed in the literature [10–12]. We find that when we add a large boson mass to the SYM theory the resulting theory is not a sensible theory unless we also subtract a logarithmically divergent mass term. Otherwise this logarithmically divergent mass term causes the spectrum to diverge logarithmically with the resolution. When we subtract the logarithmically divergent mass (renormalize it away), we find the one-parameter family of theories considered by KDB and Kutasov.

When we add a large fermion mass to the SYM theory, the resulting bosonic theory does appear to be a sensible theory. The masses are linear in $1/K$ and have a well defined continuum limit. The spectrum is very stringy, as is the spectrum of the original SYM theory. As we increase the resolution, we find states with lower masses and more partons. In comparing it to the work of KDB, we see that they renormalized away the logarithmically divergent mass term, treat the Coulomb singularity differently from the way it is treated in SDLCQ, and add a bare mass. The spectrum that they find also converges well in $1/K$, and the spectrum is very QCD-like. The low-mass states have a few valence partons, and, as one increases the resolution, one finds higher mass states. When we renormalize away the logarithmically divergent mass and add a bare mass but use the SDLCQ treatment of the Coulomb singularity, we also find a QCD-like spectrum for some values of the bare mass. If the bare mass is too small, however, we find that the spectrum is badly behaved.

In summary, adding a large fermion mass to SDLCQ produces a sensible theory while adding a large boson mass does not. The SDLCQ Hamiltonian formulation in $1+1$ dimensions is simple enough that we are able to identify the operator that needed to be added or altered to produce sensible theories. In higher dimensions there is a larger list of operators to consider and this will represent a significant challenge.

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